

Mathematics

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THE MATHEMATICS TEACHER

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« Contents »

Ability Grouping in High School	<i>Ferdinand Kertes</i>	5
Central Association Honors Dr. Slaught		16
Co-operation in Mathematics and Science		17
Reprints Now Available		21
Euclid, Agrarian Arbitrator	<i>Louise A. Forrest</i>	22
Junior High School Conference		26
The Future Geometry	<i>Barnet Rudman</i>	27
An Argument for a Correlated Course in Science and Algebra		
.....	<i>L. Ashley Rich</i>	33
Three Mathematical Songs		36
Charles Lutwidge Dodgson (Lewis Carroll)		38
News Notes		44
National Council Meeting		47
National Council Ballot		51

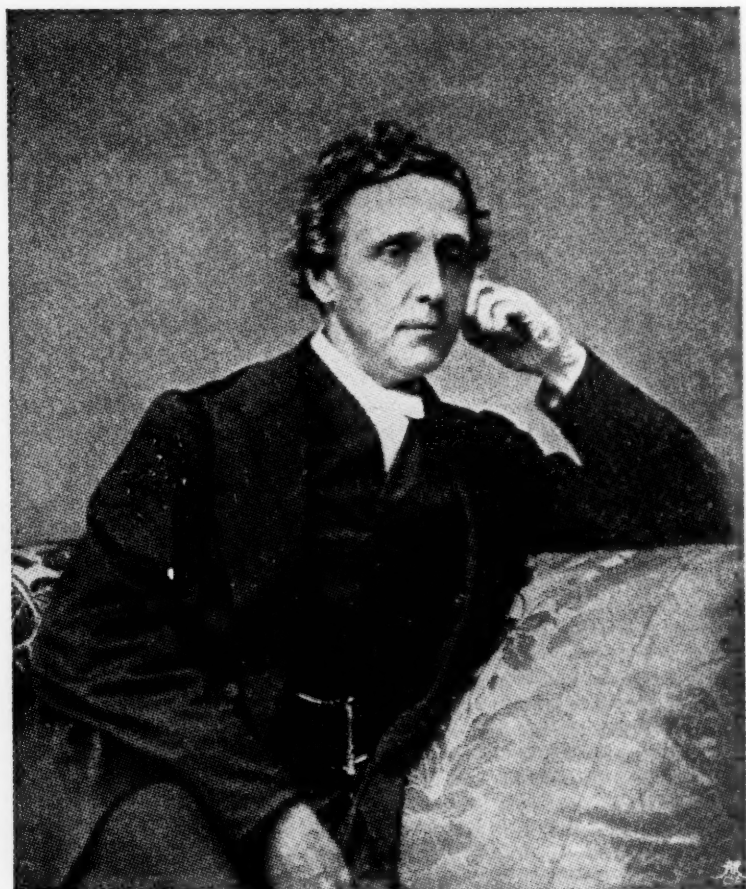
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Lewis Carroll.

THE MATHEMATICS TEACHER

Volume XXV



Number 1

Edited by William David Reeve

Ability Grouping in the High School

By FERDINAND KERTES, *Perth Amboy, New Jersey*

I. *Introductory*

It is a matter of common observation by the class-room teacher that accompanying the rapid expansion of the high school during the past decade or two, the character of the high school population has been undergoing remarkable changes. Perhaps the most outstanding feature of this change is the extension in the range of abilities, measured by any standard.

The object of this study is to determine the distribution of freshmen in the Perth Amboy High School with reference to such factors as the intelligence quotient, achievement as revealed by teachers' marks, and scores made on an algebra prognosis test; also to determine the correlation among these factors; and to determine as far as practicable, the best method of grouping pupils by ability for the purpose of facilitating instruction.

In undertaking this study, the problem of grouping pupils in the mathematics department was kept foremost in mind. Much of the data gathered can, however, be used to study pupils in other departments.

II. *Distribution*

Distribution of Freshmen by I.Q.'s

The classes of February and June, 1933, were given the Terman Group Test of Mental Ability (Form A). These tests were ad-

ministered and scored by the regular teachers of grammar and No. 11 Schools. The I.Q. of each student was then computed and sent to the high school. The distribution of pupils with reference to this one factor is shown in the following table.

TABLE 1
DISTRIBUTION OF I.Q.'s

I.Q.	Frequencies		
	Class of February 1933	Class of June 1933	Both Classes
152.5-157.5	1	0	1
147.5-152.5	0	0	0
142.5-147.5	1	0	1
137.5-142.5	1	0	1
132.5-137.5	2	2	4
127.5-132.5	7	1	8
122.5-127.5	8	7	15
117.5-122.5	8	15	23
112.5-117.5	11	15	26
107.5-112.5	19	27	46
102.5-107.5	10	32	42
97.5-102.5	30	39	69
92.5- 97.5	19	36	55
87.5- 92.5	15	27	42
82.5- 87.5	14	14	28
77.5- 82.5	8	5	13
72.5- 77.5	1	1	2
67.5- 72.5	3	0	3
62.5- 67.5	2	0	2
57.5- 62.5	1	0	1
52.5- 57.5	1	0	1
<i>Totals</i>	162	221	383

An examination of this table shows that with regard to the I.Q. factor, very little selection has taken place among the entering freshmen. Except for minor irregularities the distribution is almost normal.

In an unselected group 25 per cent of the pupils would have an I.Q. of 110 or more, 50 per cent would have an I.Q. of 90 to 110, and 25 per cent would have an I.Q. of 90 or less. (Klapper, *Contemporary Education—Its Principles and Practice*, p. 442-443). For the purpose of comparing the above pupils with a normal group, the former were grouped within these limits, with results as follows:

TABLE 2
LEVELS OF INTELLIGENCE

The I.Q.	Class: Feb. '33		Class: June '33		Both Classes		Unselected Group
	No.	Per Cent	No.	Per Cent	No.	Per Cent	Per Cent
110 or higher	49	30	54	24.4	103	27	25
90.-110	73	45	131	59.3	204	53	50
90 or lower	40	25	36	16.3	76	20	25
<i>Totals</i>	162	100	221	100	383	100	100

The figures in this table give a striking confirmation to the conclusion reached by an examination of Table 1.

If the I.Q. then, as measured by the group test of mental ability, is a highly significant measure of scholastic aptitude, it is evident that a considerable number of pupils entering the high school possess a low degree of this aptitude. The only question that remains to be settled is the validity of the I.Q., as determined by a group test of mental ability, to measure scholastic aptitude.

Distribution of Freshmen by Teachers' Marks

Teachers' marks represent another factor which may be fairly considered in estimating a pupil's ability. Of course such marks are subjective in character, but they represent an estimate of pupil ability by teachers who have had an opportunity to observe the pupil over some period of time. Considered in connection with other factors, such marks must certainly have some value.

The marks taken for this study were those in 8-A Arithmetic and 8-A Language, which represent two basic subjects. These marks were taken from the Progress Cards sent to the high school for the class of June, 1933. Out of the 221 students in this class, 214 had

marks in arithmetic and language. No grades were indicated for the remaining 7, hence these cases are omitted in the table below.

TABLE 3
DISTRIBUTION OF 8-A MARKS
Final Averages

Grades (Final Averages)	Frequencies	
	Arithmetic 8-A	Language 8-A
97.....	0	0
94.....	0	6
91.....	4	18
88.....	6	22
85.....	20	36
82.....	19	30
79.....	31	36
76.....	32	31
73.....	26	18
70.....	54	15
67.....	6	0
64.....	9	2
61.....	4	0
58.....	3	0
<i>Totals</i>	214	214

A piling of 8-A arithmetic marks near the lower end of the scale is apparent. Thus there are 54 marks in the interval 70-72.99, which is almost double the frequency in any other interval.

The contrast between achievement in arithmetic and language is striking. Of the 214 pupils, 112, or 52% have a final grade of 82% or better in language; only 49, or 23%, have a corresponding grade in arithmetic. At the lower end of the scale, 17 (8%) of the pupils have grades of 72 or less in language; 76 (36%) have corresponding grades in arithmetic. Two pupils entered the high school with grades of less than 70 in English; 22 pupils entered with corresponding grades in arithmetic.

It is evident that considering the single factor of teachers' marks in Language 8-A, the entering freshmen are a highly selected group.

But when the other factor, namely, Arithmetic 8-A, is taken into consideration, it seems apparent that an abnormal situation exists. The evidence would seem to indicate that language has been over-taught at the expense of arithmetic.

Distribution of Freshmen by Algebra Prognosis Test Scores

All of the 221 freshmen in the June, 1933, class were given the Orleans Prognosis Test in Algebra. The test was administered and the papers scored by the members of the high school faculty. The results of the test are indicated in Table 4.

TABLE 4
ORLEANS ALGEBRA PROGNOSIS TEST

Scores	Frequencies
190.....	1
180.....	0
170.....	2
160.....	1
150.....	3
140.....	8
130.....	16
120.....	22
110.....	29
100.....	23
90.....	22
80.....	27
70.....	23
60.....	16
50.....	12
40.....	9
30.....	5
20.....	2
<i>Total</i>	221

An examination of this table shows that algebraic abilities as revealed by the Orleans Prognosis Test are quite normally distributed. There is revealed no tendency for the scores to pile up at either end of the scale, as was the case with 8-A arithmetic marks.

Of the 221 students examined by the Orleans Test, 115 took the course in Algebra 9-A. The tendency indicated by the scores of these students indicates a certain amount of selection. Elimination took place in all parts of the scale, but it was much heavier at the lower end than at the upper. (See Table 5.)

TABLE 5
ILLUSTRATING TENDENCY TOWARD SELECTION OF ALGEBRA 9-A STUDENTS

Orleans Scores	Total Frequencies	Frequencies in Alg. 9-A Classes	Per Cent of Total Number Eliminated
190.....	1	1	0
180.....	0	0	0
170.....	2	1	0.5
160.....	1	1	0
150.....	3	2	0.5
140.....	8	5	1.3
130.....	16	10	2.7
120.....	22	18	1.8
110.....	29	18	5.0
100.....	23	9	6.3
90.....	22	12	4.5
80.....	27	14	5.9
70.....	23	7	7.2
60.....	16	9	3.2
50.....	12	4	3.6
40.....	9	2	3.2
30.....	5	1	1.8
20.....	2	1	0.5
<i>Totals</i>	221	115	48.0

Students at the lowest end of the scale are found among Algebra 9-A students. The range of ability among students taking Algebra 9-A is about 170 points on the Orleans Algebra Prognosis Test. This fact seems to indicate the necessity for guidance, both by means of individual counseling as to election of courses and by means of some type of ability grouping.

Evidence of Selection in the Senior Class

In order to determine whether students in the senior class are a more highly selected group, the students in the February, 1930, class were grouped in the three levels of ability indicated in Table 2. The resultant figures are compared in Table 6 with similar figures for the class of 1933, and with the corresponding percentages likely to be found in an unselected group.

TABLE 6
CONTRASTING FRESHMEN AND SENIORS WITH UNSELECTED GROUP

I.Q.	Class: 1933		Class: June '30		Unselected
	No.	Per Cent	No.	Per Cent	Per Cent
110 or higher	103	27	19	29	25
Abt. 90-110	204	53	43	65	50
90 or lower	76	20	4	6	25
Totals.....	383	100	66	100	100

The figures in this table should be taken to indicate trends rather than final percentages. With this limitation in mind, it is evident that the senior class is a much more highly selected group than the freshman class. There is a tendency to eliminate the lowest quartile altogether, and to limit graduates to pupils in the upper three quartiles.

Probably no single factor is entirely responsible for this process of elimination. Heterogeneous grouping of pupils is, however, considered to be a contributing factor. Ryan and Crecelius, who have made a thorough study of ability grouping in the junior high school, make the following statement:

Where there is heterogeneous grouping, there is one factor which constantly favors the elimination of pupils from school. No matter how many get discouraged and drop out, there is always a conspicuous tail-ender who soon is marked for the next sacrifice to "standards." He is continually so far behind the best in the class that it is easy to think of him as an exception. (*Ability Grouping in the Junior High School*, Ryan and Crecelius; Harcourt, Brace & Co. 1927. Page 9.)

Summary and Conclusions

When measured by such objective standards as the Terman Group Test of Mental Ability and the Orleans Prognosis Test, pupils enter-

ing the high school form a largely unselected group. In contrast with the students just entering the high school, those just leaving it form a considerably more highly selected group.

When measured by such subjective standards as teachers' marks in 8-A English, the freshmen entering the high school appear to be a highly selected group. The reverse is true when the same pupils are measured by their marks in 8-A Arithmetic. Since, however, these standards are subjective, they need to be examined in the light of other factors before further conclusions can be drawn from them.

When measured by any standard, students show such wide variations in ability as to justify the study of any device which promises to simplify the teacher's problem of instructing them.

III. *Ability Grouping*

Introductory

Perhaps no phase of modern education has provoked more controversy than the subject of ability grouping. Opponents of the idea argue that it is undemocratic. Proponents of the idea claim that it makes for greater economy and effectiveness in classroom instruction.

It may clarify matters if we divide ability grouping into three distinct phases, namely, grouping in the elementary school, in the junior high school, and finally in the senior high school.

This study is not intended to settle the philosophical aspects of ability grouping as such in any part of the school system. Its scope is limited to the determination of the most effective criteria for purposes of grouping by subjects in a four-year high school.

The experiences of progressive school systems which have been experimenting with ability grouping do not clearly demonstrate the desirability of ability grouping in all subjects. Furthermore, it seems to be quite generally agreed that if ability grouping is adopted, it is not of itself sufficient to materially affect the problem of instruction. Along with it must come changes in methods of teaching and the construction of differentiated courses of study for the various ability levels. A discussion of this will be found in "Vitalizing the High School Curriculum," Research Bulletin of the N.E.A., September, 1929; also in Chapter VIII, "Development of the High School Curriculum," The Sixth Yearbook of the Department of Superintendence (Vol. VII, No. 4).

It may also be pointed out that ability grouping is not the only

device being investigated for the purpose of finding a satisfactory solution to the problem created by the rapid growth and changing character of the high school population. Among other devices under consideration may be mentioned the enrichment and the revision of the entire curriculum, accompanied by more effective methods of guidance, and the individualization of instruction. The latter is especially frequent in the more technical fields of the curriculum.

Reports on Ability Grouping in Mathematics

A plan adopted by a number of school systems is reported on Page 217 of the Research Bulletin of the N.E.A., "Vitalizing the High School Curriculum."

All students who register for the first course in elementary mathematics are classified in Groups I, II, or III, according to their ratings in arithmetic based upon the eighth grade teachers' judgment and upon their achievement quotient sent up from the grades. If these have not been furnished, their ratings shall be determined by some mathematics test given under the direction of the head of the department. The program schedule should be arranged so that a I, II, and III group can meet at the same hour. This will make it possible to have a student shift from one group to the other without disorganizing his whole program, hence errors in classification may be adjusted early and the student may be kept up to his true standard of ability in mathematics. At the close of the term, the head of the department can arrange the students in ability groups from the records of the current and previous terms.

"An Experiment in Grouping Pupils According to Ability in High School Algebra" is summarized on pp. 330-331 of the Sixth Yearbook of the Department of Superintendence on "The Development of the High School Curriculum." A summary of the findings is as follows:

1. A ranking based upon a combination of teachers' marks in the first semester of algebra and the intelligence quotient is the most effective method of classification.
2. Teachers' marks are slightly better than gross scores on tests.
3. The intelligence quotient alone is about equal in value to the gross scores. (The results of the present study do not bear out this assertion.)
4. The arithmetic sections of the Terman Test are about as good as the whole test for purposes of grouping.
5. Probably the best way to group pupils for the first semester algebra would be to use a composite of a score based on eighth-grade arithmetic and the intelligence quotient. (The results of the present study indicate that best results are obtained by using a composite of a score based on eighth grade arithmetic and the Orleans Prognostic Test of Algebra. This difference of opinion is probably due to the fact that at the time the conclusion in Section 5 above was reached, the Orleans Prognostic Test in Algebra was not yet available.)

At a recent meeting of the New Jersey Mathematics Teachers' Association there was a report on Ability Grouping in the Dickinson High School, Jersey City, based on the results of the Orleans Prognosis Test in Algebra. The reported correlation of approximately 0.65 between the prognostic test scores and final averages in algebra was said to be considerably higher than the correlation between I.Q.'s and algebra marks.

Mr. Carlson, principal of the New Brunswick High School, in a recent talk to the class in high school administration at Rutgers University, reported quite satisfactory results from a method of grouping based upon the I.Q., teachers' judgments, and a retardation factor taking into consideration the chronological age of the pupil in relation to his placement.

All of these reports agree in one respect, namely, a tendency toward continuous experimentation to determine the best method of grouping. It seems to be the consensus of opinion based upon the experiments thus far worked out that no single factor can give a best method of grouping. The best method of grouping seems to be based upon a composite score made up of a number of different factors.

Grouping Pupils by I.Q.'s

Students in the classes of February and June, 1933, were grouped by their intelligence quotients in the first semester's work in algebra. At the end of each term final averages in the first semester's work were correlated with the I.Q.'s. The coefficient of correlation based upon 219 cases was found to be .56 with a probable error of 0.037.

Investigation of Other Possible Methods of Grouping

For the purpose of determining whether a prognostic test in algebra would yield a more reliable basis for grouping, students in the June, 1933, class were given the Orleans Algebra Prognosis Test. The test scores were not made the basis of grouping, but at the end of the first term's work they were correlated with the final averages in algebra. The resulting coefficient of correlation, based on 114 cases, was 0.61, with a probable error of 0.04.

Taking the algebra marks of the same pupils, and comparing them with their I.Q.'s, the coefficient of correlation was found to be 0.50, with a probable error of 0.047.

A comparison between the two correlations will show that the

Orleans test is somewhat more reliable than the I.Q. for the purpose of grouping pupils in algebra.

Teachers' judgments of the same pupils, as revealed by final averages in Arithmetic 8-A, were similarly correlated with algebra marks. The coefficient of correlation was here found to be 0.63, with a probable error of 0.038.

Ability grouping based upon arithmetic marks is thus found to be superior to grouping by I.Q.'s and approximately as reliable as grouping by algebra prognosis test scores.

Composite Scores as Bases for Ability Grouping.

Composite scores for the following were obtained by the method outlined in McCall's *How to Measure in Education*, Chap. II, pp. 19-66. The multipliers used in obtaining individual scores were 0.6 for the Orleans scores and 0.8 for the I.Q.'s.

1. I.Q.'s and Arithmetic 8-A marks.
2. I.Q.'s and Orleans test scores.
3. Orleans test scores and Arithmetic 8-A marks.
4. I.Q.'s Orleans test scores and Arithmetic 8-A marks.

Each series of composite scores was then correlated with final averages for the first semester's work in algebra. The results are indicated in the table below:

TABLE 7
TABLE OF CORRELATION COEFFICIENTS

Composite Scores	Correlation with Algebra Marks	No. of Cases	Probable Error
1. I.Q. & Ar.	0.66	112	0.038
2. I.Q. & Or. Sc.	0.68	112	0.034
3. Or. Sc. & Ar.	0.72	112	0.031
4. I.Q., Or. Sc., Ar.	0.68	112	0.034

Limitations to Significance of Results

The results of this study do not prove that ability grouping will make instruction more effective or reduce the number of failures. They do indicate, however, that certain criteria predict, with a fair degree of accuracy, ability in freshman algebra. Criteria in other fields can presumably be determined by similar methods. Furthermore, certain criteria have been found to be more effective than

others. But these criteria could be used for purposes of guidance as well as grouping by ability.

It must not be forgotten that the correlations obtained are based partially on subjective standards. Hence the reliability of the results is limited by the reliability of the subjective standards.

It should also be borne in mind that the results obtained are at best indicative of mass tendencies only. They do not preclude the necessity of carefully studying individual cases. On the contrary, since the correlations are not perfect, there will be individual cases of students who are exceptions to the general rule. Grouping, if adopted, should therefore be flexible, so as to allow readjustments when necessary.

Conclusions

1. The final averages in Arithmetic 8-A are as valuable as any single factor for the purpose of grouping in freshman algebra.
2. Composite scores in any form are more reliable indications of ability for purposes of grouping than single scores.
3. A composite score made up of final averages in Arithmetic 8-A and the scores obtained by the use of the Orleans Prognosis Test in Algebra, is the best index obtained for purposes of grouping in first semester algebra.
4. The I.Q. alone shows a tendency to be the least reliable index for grouping in algebra.

Central Association Honors Dr. Slaughter

WHEREAS, Herbert Ellsworth Slaughter, Professor of Mathematics, University of Chicago, has shown a deep and continued interest in the Central Association for more than a quarter of a century, appearing on our programs from time to time, Vice-chairman and chairman of the Mathematics Section, 1906-1907, publishing articles in our official journal, *School Science and Mathematics*, and whereas he is revered as one of America's leading educators in his chosen field, a text-book writer of national reputation and influence, a Past President of the Mathematical Association of America, be it therefore

Resolved, That the Officers and Board of Directors of the Central Association delight to honor Professor Slaughter by electing him an Honorary Member of this Association.

Co-operation in Mathematics and Science*

ONE OF THE most interesting developments of modern science has been its widespread demand for mathematical assistance. Biology, medicine, finance, economics and many others are calling on mathematics for help in solving their problems. Pre-professional courses in our colleges show a continuously increasing demand for it directly and through physics and chemistry. The student who can not use his arithmetic and elementary algebra easily and well has little or no chance to succeed in them.

The Departments of Physics and Chemistry of the University of Pennsylvania, with the co-operation of the Department of Mathematics, are insisting that students who take their courses shall have an efficient facility in using the elementary numerical and algebraic processes necessary to handle their problems. These are details which the student usually acquires in grade and preparatory schools. Each student on entering one of their beginning courses is given a list of such processes used in that course. He is told that for the course he is then entering an easy, quick, intelligent use of them is necessary and is required. If he proves deficient in them, he is required to do outside work in them until he shows a reasonable proficiency; and this must be done within a definite time limit.

I. A copy of the following is handed to each student beginning physics.

A Command of School Mathematics Necessary for the Study of Physics.

A student who is beginning the study of physics in college should realize that he will have immediate and constant use for all the mathematics he has learned in school—arithmetic, algebra, geometry and plane trigonometry. Lack of a ready working knowledge

* The material found in this article represents the results of the deliberation of a joint committee from the Departments of Mathematics, Physics and Chemistry of the University of Pennsylvania. The Committee was appointed to discuss the lack of mathematical facility exhibited by students in the elementary college courses in mathematics and science. Professor H. B. Evans of the Department of Mathematics sent us the material.—THE EDITOR.

of these school subjects is a most frequent cause of waste of time, waste of money, disappointment and ultimate failure.

Do you possess this ready working knowledge? Can you without seeking aid solve the problems attached to this paper in two hours? If not, you are advised to postpone the study of college physics until you can do so. Otherwise you will find yourself spending all the time needed for the study of physics in attempting to learn the mathematics that you should have mastered in school.

As a time and labor saver you will need a simple slide rule. An adequate slide rule can be had for one dollar. Get one, master it, and use it in performing numerical calculations. Use the hours saved by its use in learning the facts and principles of physics.

PROBLEMS

- Express as decimal fractions $1/40$, $7/8$, $9/25$, $16/125$.
- Find the sum of: two tenths,
three fortieths,
seventeen thousandths, and
six ten thousandths.
- Find R to three figures.

$$R = \frac{1.982}{2} + \frac{[5.02]^2}{2 \times 1.982}$$

(Do not "clear of fractions," or "reduce to a common denominator.")

- Find M_0 to five figures.

$$M_0 = 21.972 + (21.975 - 21.972) \frac{12.2 - 10.8}{12.2 - 8.7}$$

(Do not "clear of fractions.")

-

$$Y = \frac{\frac{50 \times 980}{\pi \times (0.021)^2}}{\frac{0.069}{196}}$$

Solve for Y in the form $K \times 10^{11}$, K a number of two digits.

- One hundred and twenty rotations per minute is how many rotations per second? per hour?
- a , b , d , and ω are known.
 $Wx = \omega a$ and $W(x+d) = \omega b$ are known relations.

Show that

$$W = \frac{\omega(b-a)}{d} \text{ and } x = \frac{ad}{b-a}$$

8. $100 = 80t - 16t^2$.

Find t .

9. A man travels 15 miles in an easterly direction and then 20 miles in a northeasterly direction. 1. How far is he from his starting point? 2. How far east, and how far north from his starting point?

10. A man walks 10 miles north, then 14.1 miles south-east, and finally 5 miles west. How far and in what direction is he then from his starting point? Check by a diagram.

11.

$$\frac{8}{17}F - F \cos \Theta = 0$$

$$\frac{6}{17}F + F \sin \Theta = 84$$

Find F and Θ .

12. Find the circumference and area of a circle one foot in diameter.

13. Find the area of a triangle whose sides are 20, 20, and 24 feet.

14. Find the surface area and volume of a sphere one foot in diameter.

15. A wire of uniform circular cross-section, 120 centimeters long has a volume of 10.0 cubic centimeters. Find its cross-section and radius.

16. Which is running the faster a man running 100 yards in 10 seconds, or a horse running a mile in 3 minutes?

17. Which is the greater speed 100 meters in 11 seconds, or 25 miles per hour?

18. Make a table of the sines, cosines and tangents of 0° , 30° , 45° , 60° , 90° .

II. A copy of the following is handed to each student beginning chemistry.

Chemistry I.

Students registering for Chemistry I must be able to work problems of the following types *with facility*. Test your ability by solv-

ing each problem. Answers are placed at the end of the list. The average student should complete this exercise in 90 minutes.

All students in Chemistry I must provide themselves with a slide-rule (a satisfactory slide-rule may be purchased for one dollar), and be able to use it for the following operations: multiplication, division, squaring numbers, extracting square roots, finding logarithms and antilogarithms.

Students must be familiar with the units of length, volume and mass as expressed in the Metric System.

1. $0.2 \div 500 =$.
2. $400 \div 0.8 =$.
3. Express 7.52% as a decimal.
4. Express 0.00234 as %.
5. Convert $\frac{13}{17}$ to a decimal.
6. Subtract (-32) from 18.
7. Subtract 32 from -18 .
8. Add 273 to -60 .
9. Log 2345 = .
10. Log 0.000123 = .
11. Log $\frac{1}{1.43} =$.
12. Log $2.3 \times 10^7 =$.
13. Log $\frac{1}{2.3 \times 10^{-7}} =$.
14. Antilog 1.6990 = .
15. Antilog $8.5933 - 10 =$.
16. $16^{2.3} =$.
17. $\sqrt{12345} =$.
18. $\sqrt[3]{12345} =$.
19. $(54321)^{1/5} =$.
20. Express 12300 in the form $a 10^k$ where k is an integer and a is a number between 0 and 10.
21. Express 0.000023 in the form $a 10^k$ where k is an integer and a is a number between 0 and 10.
22. Convert 1.23×10^3 to a whole number.
23. Express 4.52×10^{-3} as a decimal.
24. Add 3.0×10^{-3} to 4.0×10^{-4} .
25. Multiply 4.5×10^{-3} by 6.6×10^3 .
26. $\frac{6.4 \times 10^{-11}}{2.5 \times 10^{-3}} =$.
27. $(5.6 \times 10^{-3})^2 =$.
28. $\sqrt{1.8 \times 10^{-3}} =$.
29. $\sqrt[3]{0.8 \times 10^{-3}} =$.
30. $4A^3 = 1.6 \times 10^{-49}$. Solve for A .
31. Solve $3(+2) + 2A + 8(-2) = 0$ for A .
32. Solve $2(+3) + 3A = 0$ for A .
33. Solve $4:50 = 1.5:A$ for A .
34. Solve $\frac{5 \times 0.4}{6} = \frac{7 \times 0.8}{A}$ for A .
35. Solve $\frac{1}{2} = 9 \times 0.5 \times A$ for A .
36. Solve $\frac{5 \times 0.6}{\frac{A}{185}} = 2$ for A .
37. Solve $\frac{270}{\frac{A}{5}} = 4$ for A .
38. Solve $\frac{1.5 \times 0.6}{\frac{6}{A}} = \frac{1}{4}$ for A .
39. $\frac{A}{5}$ is inversely proportional to $\frac{1}{4}$. Solve for A .
40. Add $b(1-a) + ba + 2ba =$
41. Solve $1.45 = \frac{(1+2A) 33.3 \times 1.86}{111}$ for A .

ANSWERS

1. 0.0004	15. 0.0392	29. 2.0×10^{-3}
2. 500	16. 588.1	30. $A = 3.4 \times 10^{-17}$
3. 0.0752	17. 111.1	31. $A = +5$
4. 0.234%	18. 23.11	32. $A = -2$
5. 0.7647	19. 6138	33. $A = 18.75$
6. 50	20. 1.23×10^4	34. $A = 16.8$
7. -50	21. 2.3×10^{-5}	35. $A = 0.111$
8. 213	22. 1230	36. $A = 0.0081$
9. 3.3701	23. 0.00452	37. $A = 337.5$
10. 6.0899-10	24. 3.4×10^{-3}	38. $A = 1.67$
11. 9.8447-10	25. 2.97×10^6	39. $A = 20$
12. 7.3617	26. 2.56×10^{-8}	40. $b(1+2a)$
13. 6.64	27. 3.14×10^{-5}	41. $A = 0.799$
14. 50	28. 4.24×10^{-3}	

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THE MATHEMATICS TEACHER
525 W. 120th Street, New York, N.Y.

Euclid, Agrarian Arbitrator

By LOUISE A. FORREST

Roxbury Memorial High School, Boston, Massachusetts

IN THE EFFORT to establish congruence theorems as stock-in-trade for the pupils of the tenth grade, probably most teachers have vacillated between accepting the modern recommendation of informal demonstration and the old idea of building a logical structure from the outset by means of rigid proof. No one questions the difficulty of making children understand proof by "placing," especially in the early part of the first year of demonstrative geometry, but in spite of this, many teachers would like to conclude these theorems satisfactorily in their proper sequence.

In an attempt to do this and yet take into account the perplexities of the immature mind, the following sketch was worked out, seemingly successfully. The part of the defendant was played in a convincing way by a pupil who had completed a year of geometry. The other members of the cast and the jury were selected from the tenth grade class. The verdict brought in was entirely honest and was unanimously in favor of congruence.

After the court adjourned the class discussed the piece and decided to accept the general fact of congruence of triangles by two sides and the included angle. Better than that, impersonating lawyers, they experimented with triangles in which two angles and the included side of one were known to be equal to two angles and the included side of the other and succeeded in proving them congruent with none of the usual incredulity apparent. All this was accomplished in a class period of forty-two minutes.

Scene—Court-room with table and chair, gavel, etc., for judge, chairs for plaintiff and defendant, etc.

CLERK *enters, dressed in academic gown*

CLERK. Hear ye! Hear ye! This honorable court is now in session pursuant to adjournment. Govern yourselves accordingly. (*People stand while judge, dressed in academic gown, enters and takes his seat.*)

JUDGE. Mr. Clerk, what is the first case?

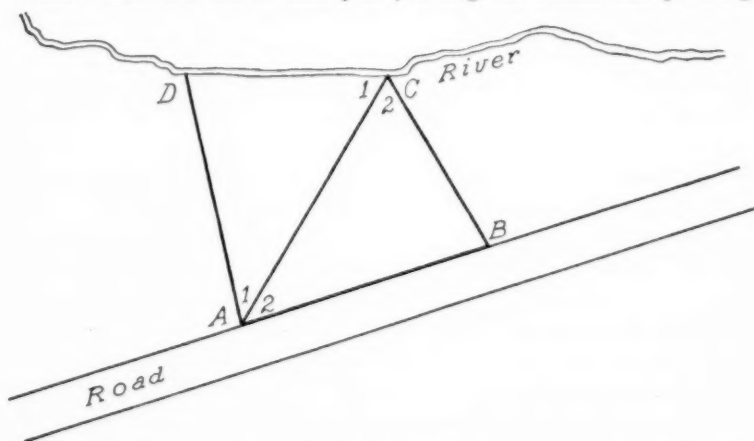
CLERK. Vanya *vs.* Government.

JUDGE. Mr. Clerk, bring in a jury.

CLERK. The jury has been selected and is present, Your Honor. Will the jury please stand. (*Jury, which has previously been selected but which knows nothing about the case, stands for an instant, then sits.*)

JUDGE. Mr. Clerk, we shall now hear the first case.

PLAINTIFF (*dressed in Russian smock*). Your Honor, I am a poor farmer. My home is outside Vladivostok near the river. All my life I have tilled the soil in that place, raking out stones and planting



wheat. But there were many stones and the wheat has not been full and strong. I have worked very hard and I am poor. I sell my wheat for only a few kopecks, hardly enough to support my family. And now there comes this man, who says he must buy half my land. And if I will not sell it, he must buy it anyway. The government will take it from me. He says he will pay. All right, I am willing. But he puts two stakes in the ground and ties a rope between and says: On this side, your land; here, mine. And I know he takes more than half my land. True, he gives me the land by the river but he gives me none by the road. I wouldn't mind if I didn't think he takes the lion's share. And when I object, he sits on a rock and draws pictures of things which he calls angles and explains until my head is weary. I am sure that I understand about his angles, but he will not measure all around the fields.

DEFENDANT. No, no, I will not measure all around the fields. When

I know the length of two sides of a field of this sort, and the size of the angle between those sides, then I know whether or not one field is as big as another. Now, Your Honor, the farm looks like this. It goes from A to B , to C , to D and back to A again. (*He produces a map.*)

The side CB is as long as the side DC .

PLAINTIFF. How long?

JUDGE (*rapping for order*). Order! My man, you must not interrupt the court in this way.

PLAINTIFF. But, Your Honor, I would like to know how long that boundary is.

DEFENDANT. I'll tell you. Both CB and DC are 195 meters. But the number doesn't matter. What matters is that their lengths are the same. Also angle C_1 and angle C_2 are equal.

PLAINTIFF. I object, Your Honor. How does this lawyer know the sizes of these boundary lines and angles?

JUDGE. Objection sustained. (*To lawyer*) Please explain.

DEFENDANT. I procured my information from the Registry of Deeds of Vladivostok. In that office are recorded the official measurements of all farms in this neighborhood. Now, the rope that is strung from C to A will be one length whether we consider it the boundary of my field or of his. Then behold! the field I leave him, ADC , is as big as the field I take, ABC .

PLAINTIFF. If he would measure every boundary line and every corner angle, I should be content. But he will not. And so I come to you, honorable sir, for justice.

DEFENDANT. I can show you that it is enough to know the parts which I have described. Nowhere in all the world would an educated man do the unnecessary work which this poor farmer asks. Twenty-two hundred years ago in ancient Greece men saw that it was sufficient to know three parts of a triangle to know its size. Now these lots are triangular in shape, Your Honor, and if a man has studied just a little bit of Euclid, he knows that a triangle has six parts that can be measured—three angles and three sides—and he knows that if he chooses wisely, information about three of the six parts is enough. I will show you that what I say is true. Take two triangles such as these. Can you fancy that triangle ABC be folded over on the line AC , so that it lies somewhere on the other field? Is not the very best way for comparing the sizes of surfaces to lay them one on the other

to see if they fit? If this farmer cannot imagine my field laid over upon his, let him think of a great stiff canvas exactly the shape and size of my field spread out on his. If we place this canvas with the side AC of my field lying upon the side AC of his and let the rest of it go wherever it will, we can examine the edges and find out whether or not the canvas copy of triangle ABC fits on triangle ACD . In this diagram I shall indicate the canvas by a red line.

Your Honor, I have found in the records that angles C_1 and C_2 are equal. If angle C_2 were bigger than angle C_1 , what direction would CB take? It would go off here to the north crossing the river. (*Shows it.*)

If angle C_2 were smaller than C_1 , where would CB go? It would start from C to run off this way toward the south, and lie somewhere on this side of the river. (*Shows it.*) But the opening of angle C_2 is exactly the same as the opening of angle C_1 . Then must not CB take the direction of CD ? (*Lawyer draws a short red line along CD starting from C and going only part of the way to D .*) If you are now convinced that the canvas replica of my field when placed on this farmer's field would have the edge CB run along the edge CD , the next thing to find out is how far it runs. If my field is bigger than field ACD , should we not expect the corner B to be farther along the river than D ? But it isn't farther along than D . B is exactly where D is, for after I found that CB ran along CD I examined both lines, CB and CD , in the recorded deed and found them of equal length. Don't you see then that the point B must lie on D ?

So far, you agree that triangle ABC fits on triangle ACD throughout the length of CA , around the corner C , and throughout the length of CD ? Then there is no more work to do. Anyone knows that if a stake is placed at D (where B now is) and another at A , that only one straight line can be drawn between them. And mercifully, both boundary lines BA and DA are straight. Therefore, the straight boundary line DA stretched between D and A must be exactly where the straight line BA is, for it, too, is stretched from D (where B now is) to A .

Honorable Sir, I have examined only three parts of each triangle, two sides and the included angle in each case. In triangle ABC , I examined the lengths of AC and CB and the size of angle C_2 , but I did not bother to find out the values of angles B or A_2 or of line BA . In triangle ACD , likewise, I examined only the lengths of sides AC and CD and the size of angle C_1 .

But I have shown you that even without knowing the measurements of the other parts, that the first triangle must fit on the second. Would it not be wasting my valuable time to study six parts when I wish to compare the sizes of two triangles? I know that triangles will fit exactly, or, as we say, coincide, when I know that two sides and the included angle of one are equal respectively to two sides and the included angle of the other. I rest my case.

JUDGE (*addressing jury*). Gentlemen of the jury, I submit the case to you. If you find that the farmer has been given less than his fair share of the land, then bring in your verdict in his favor. But if you find that the government agent has shown that these two triangular lots are of equal size, then you must bring in a verdict for the government. Will you please retire to the adjoining room and decide upon your verdict and report as soon as possible.

(*Recess while jury is out*)

(*Jury returns and reports verdict*)

JUDGE. The court is now adjourned.

Junior High School Conference

The Eighth Annual Junior-High-School Conference will be held at New York University on March 18-19, 1932. The central theme of this conference will be: "Improving Junior-High-School Instruction." Two general sessions coming Friday evening and Saturday morning will be followed by some thirty Round Tables related to the central topic.

This conference is unique in that it is a co-operative arrangement, the directive force being vested in an Advisory Committee of thirty-six representative educators distributed throughout the eastern states. The committee just referred to gives practical direction to the character of the conference. New York University offers the facilities for making this clearing house treatment of junior high school problems possible.

The regional character of the conference makes it one of the most important of its kind in the country. In 1931, upwards of 2,000 attended it. The following states took an active part in the conference as revealed by talent and individual attendance: New York, Pennsylvania, New Jersey, Connecticut, Delaware, Maryland, Rhode Island, Massachusetts, and Ohio. It is also interesting to note that the conference has had a remarkable growth. With seven round tables and thirty speakers in 1925, it has expanded to include thirty-one round tables and one hundred and thirty-four speakers for 1931.

The Future Geometry

By BARNET RUDMAN, *South Boston High School,
Boston, Massachusetts*

PERHAPS no educational doctrine has crystallized more definitely from the intense research of the current century than the theory that there is little or no automatic transfer of ability from one field of mental activity to another. Nothing, indeed, was more instrumental in causing the very foundation to quake and convulse under the old philosophy of education. We may not like the finality with which the new doctrine is urged upon us, we can certainly point to many defects in the case against automatic transfer, but the fight is still on, and the last shot is yet to be fired.

That the future of high school mathematics will be vitally affected by the outcome of the controversy need hardly be asserted here. For centuries geometry appealed to the intellectual curiosity of man and, because of its insistent call for pure reason, came to be regarded as a major stimulant and builder of the human mind. With the arrival of the modern high school plane geometry was given a place of prominence as one of the mainstays of the curriculum. In recent years the movement to render secondary education materially useful brought geometry, too, under the ax and, as a consequence, many so-called practical problems have been included in the modern course. Now, it is an open question whether the ability to make a scale drawing of the top of a table would prove more useful to the boy destined to sell insurance than his ability to construct a line parallel to another line; or whether the knowledge of how to compute the contents of her coal bin would be more helpful to the girl student headed for a secretaryship than a knowledge of the properties of proportions. The ultra-utilitarian holds that they are all equally useless. But, granted the utilitarian nature of the practical applications of geometry, three quarters of the course—and this is a conservative estimate—is still devoted to demonstrative work. For this part of geometry there seems to be no justification except in its potency as a means of enhancing and cultivating reflective thinking in general. To quote Professor Raleigh Schorling (from a "Report by the National Committee on Mathematical Requirements"):

"... It is also quite clear that the time and money spent on Plane Geometry cannot be justified on the basis of social utility. It must be defended for its contribution to a systematic technique of thinking or it must be omitted."

How damaging, then, to our cause the truth—if it indeed be the truth—that thinking ability does not carry over from geometry to non-geometric situations.

The new theory as formulated, since the overthrow of the Faculty Psychology, by the more moderate philosophers of education declares:

(1) No transfer of training takes place automatically to any appreciable degree.

(2) If transfer is desired, it must be deliberately planned for and made a definite part of the teaching process.

(3) Conditions must be favorable for transfer, that is, there must be common elements between the situation where the pupil learns and the situation to which the transfer is to be made.

(4) The pupil must see the existence of these common elements and understand that it is to his advantage to deal with real life problems as he dealt with the more abstract book problems.

That the above doctrine carries grave implications for us geometry teachers is quite obvious. Nothing short of a complete revolution in both teaching methods and subject matter would be adequate, if we were to teach reflective thinking by the dictates of the new theory. Propositions would have to be taught with an eye on the realities of life to the end of impressing on students that objective and systematic search for truth might be as feasible in one as it is in the other. Social, economic and domestic problems would be disentangled and analyzed in the classroom in much the same way as geometry originals and solved by the same methods of rigid adherence to logic. The very word "geometry" might have to give way to a newer name capable of more adequately embracing the extended functions of the course.

Such revolutionary measures, to be sure, are not imminent in the near future. Those that shape the destinies of high school mathematics are conservative and will not permit sweeping radical changes in the curriculum. Nor will sophisticated men of affairs subscribe to the notion that mere pedagogues and mathematicians are competent to teach the young how to think about real life. "Of what use," they will ask, "are mathematical methods in practical problems?" And they will shrug their shoulders, if it be ventured in re-

ply, with a fair degree of plausibility, that to the extent that precise scientific methods are not being used in the solving of practical problems the solutions are bungled and often result in miscarriage. So while it is unlikely that geometry will emerge intact from under the pressure of the New Psychology it is equally unlikely that extremists will be permitted to tamper with it.

Should, however, the gospel continue to spread that geometry as taught today does not make for better thinking in general, the subject would have to change both in content and in method in order to retain its present position in the high school curriculum. Something, indeed, may evolve that would bear only a distant relationship to the old Euclidean Geometry. It is conceivable that the geometry of tomorrow might include a new type of proposition, for example:

THEOREM: People should buy within their means even when buying on the installment plan.

Given: A family of five—father, mother, and three children—enjoying a very satisfactory income (about \$60 a week), paying \$55 a month rent and otherwise living a comfortable life. The sales agency of a well known automobile in the \$1,500-\$2,000 class is urging on the family what looks like an attractive trade-in on the yet serviceable four-cylinder car.

Prove: It is unwise for the family to make the change.

Analysis

A major domestic venture of this sort, the purchase of a new expensive luxury, should not be embarked upon, if it appears that:

- (1) It would work an undue strain on the family budget.
- (2) It would drain the reserve funds of the family to the extent of imperiling its security.

It can be proved that the proposed transaction threatens the family with both (1) and (2).

PROOF

Statements

1. A bigger car brings with it a substantial increase in the expenditures of the household.

Reasons

1. Insurance, interest on loaned money, running expenses, repairs, new parts, depreciation are in the long run about twice as expensive as the same items for the smaller car.

2. And this results in an undue strain on the family resources.
3. Lack of economic security to the individual and his family is a major curse of this industrial civilization.
4. A reserve fund is the only insurance against this economic insecurity and against such other emergencies as are likely to spring from a complex life.
5. The reserve fund of the family in question would be sadly depleted by the initial and subsequent payments on the bigger car.
6. The security of the household would thus be jeopardized by those whose duty it is to safeguard it.
7. \therefore It is unwise for the family to make the change.
2. The family has already been living close to its means (inferred from Given).
3. Economically the individual today is at the mercy of a pitiless industrial machine that would not hesitate to crush him beneath its wheels, if it so suits its purpose.
4. It enables a man to hold over bad times or find a new orientation, if his was the misfortune to have been in the path of the industrial steam-roller.
5. About \$1,200 or more to be paid out on the car during the first year.
6. If the father should lose his position or become incapacitated, the family would suddenly find itself helpless and destitute.

Q. E. D.

Other elements, of course, might have been included in the above demonstration. Elements like jealousy and desire for ostentation, that in many cases would swing the pendulum towards the hazardous undertaking, might have been included and reduced to absurdity when studied together with the more rational objectives in life.

Similarly definite contributions could be made to the moral education of high school pupils through a series of theorems formulated for the purpose. For instance:

THEOREM: *To participate in gambling pools and outlawed lotteries is unpatriotic, unprofitable, and detrimental to a person's character.*

Given: A boy tempted to buy a "chance" in a baseball pool out of his weekly allowance for spending money.

Prove: That, yielding to the temptation, the boy would be guilty of an act that is unpatriotic, unprofitable, and detrimental to his character.

PROOF

Statements

Reasons

- | | |
|--|----------------------|
| 1. Pools and lotteries are outlawed in this country because they are deemed immoral. | 1. Common knowledge. |
|--|----------------------|

2. By buying a pool ticket one shows disrespect for law and becomes, in fact, an accessory to a criminal act.
3. To the extent, therefore, that law abiding citizens patronize these extra-legal institutions they encourage and maintain the many "rackets" that are becoming a serious menace to organized society.
4. The chance of winning is so small as to amount to no chance at all.
5. Investments in such and similar schemes, however small, loom big when compared with the infinitesimal chance they have in bringing back the coveted fortune.
6. By engaging in gambling the boy learns to rely for success on chance rather than achievement. He acquires, too, the antisocial trait of expecting gains from sources other than faithful service to society.
7. The expectancy of sudden riches tends to create false illusions, provides food for day-dreaming, and otherwise reduces the boy's normal efficiency.
2. Axiomatic.
3. Axiomatic.
4. For every one that wins there are thousands that lose, and any sensible person knows that in all likelihood he would be among the thousands of losers, not the one winner.
5. The wisdom of an investment is measured by the probability there is in it for profit making.
6. Definition of gambling also the axiom that gambling like other vices seldom ends in anything less than addiction.
7. Adolescents are susceptible to day-dreaming, and a pool ticket in the boy's pocket would only lend some plausibility to his dreams thus deflecting his mind and energy from the real tasks of life.

It is readily conceded that each of the theorems just demonstrated lacks some of the earmarks of a geometry proof. Without access to fundamental "life" axioms, generally accepted as such, or previously established economic and social truths one necessarily faces a serious handicap. At best human problems are only moderately susceptible to rigid geometric treatment. What human truth or man-made law is endowed with the indisputability of a mathematical axiom, postulate or demonstrated theorem? The writer certainly is not guilty of the presumption that he can bring mathematical order into the chaos of human affairs. It is merely ventured in this paper that, should the hostilities against "automatic transfer" continue as bitterly as in the past two or three decades, it might some day be necessary to modify the Euclidean Geometry so as to bridge the gap between it and the

reflective thinking done in life. Within reasonable bounds the new geometry would seek to make geometric methodology applicable to human problems. True, mathematical objectivity may never carry over into law, commerce, or social relationship but the desire for objectivity could be made quite transferable.

Even today the teacher with a leaning to the newer psychology could dip into life for examples on inductive reasoning and teach at once both intuitive geometry and generalization. He might even teach a health lesson in the process. For instance: John got a slight cut while working in the machine shop and paid no attention to it. Two days later he was in the hospital with a bad case of infection. Edward stepped on a rusty nail resulting in a scratch too slight to cause him any concern. A few days later his life was barely saved by the amputation of the injured leg. The son of a recent President of the United States lost his life because of his failure to give prompt treatment to a minor bruise. The generalization is inevitable that cuts and bruises, be they ever so slight, are dangerous and must be attended to promptly and intelligently.

And from this back to geometry. James drew a small 60° - 30° right triangle and found the side opposite the 30° angle to be about one half the hypotenuse. Mary drew a 60° - 30° right triangle of medium size and, strangely enough, discovered exactly what James had previously found. Vincent now goes to the board and draws a large triangle of the same description and, sure enough, the smaller arm is still one half the hypotenuse. So the generalization is formulated that in any 60° - 30° right triangle the side opposite the 30° angle is one half the hypotenuse.

It is improbable that man will discard geometry as an educative tool. Rather it seems likely that he will seek to make it more educative. Geometry, indeed, could be charged with the task of injecting into man's thinking and life's problem-solving the very elements that are sadly missing therefrom. Some day man may decide to purge his thinking processes of the prejudice and bigotry and intellectual dishonesty that have distorted his vision, and substitute in their stead objectivity, honesty, dispassionate inquiry, open-mindedness, and steadfast refusal to accept for truth nothing but the truth. And he may ask geometry to provide the pattern for his new mode of thinking.

An Argument for a Correlated Course in Science and Algebra

By L. ASHLEY RICH

*Former Head of the Chemical Engineering Department,
Y.M.C.A. Institute of Technology, Buffalo, New York*

THE FIRST-YEAR course in algebra can be the most interesting subject in the curriculum. It would be if it were not devitalized by some of these logical minded college professors who write our texts. Any student sufficiently endowed with mental equipment to enable him to do successfully sixth grade arithmetic would encounter no difficulty in mastering most of the material in the first year algebra if it were properly presented. Yet the percentage failure of algebra students exceeds that for all other subjects in most high schools, and in addition to this it is the exceptional student who at the end of the course in June can demonstrate any true mathematical power. By mathematical power I mean ability to apply mathematical methods to new situations, possession of confidence in one's results, etc. By the following September nearly all the skills in manipulation and most of the problem work have apparently been forgotten.

Few pupils have an algebra teacher who can convince them that the juggling of abstract symbols is of any benefit to them, utilitarian or otherwise. A vote taken in several high schools showed algebra to be the most cordially disliked subject in the curriculum. Is this to be wondered at when we find such far fetched and ludicrous attempts to apply algebra to the number of marbles Johnie has. In the algebra books now in use no defense is given for the interminable lists of manipulation problems. No apparent use is made of the skills supposed to have been acquired by the student who has completed these drills. While necessary habits may be acquired by this work these drills would be more effective if a few problems were inserted that did not require exactly the same mechanical process as problem one. Does pushing button number four and turning crank seventeen constitute algebra? I dare say that not as much progress will be made by a class the first week that rote learning is discouraged and thinking substituted, but in the long run it will pay teachers to require students to think.

One has but to teach engineering classes in a technical college to see how little power is acquired by most high school mathematics students. The joke of the situation is that the trouble college students have are with algebra fundamentals such as signs, and not with the higher branches of the work. College instructors wish high schools would teach less material and teach it more thoroughly.

Is not algebra teaching a salesman's job in creating a desire on the part of the student? Can we not select topics that appeal to a student because they enable him to achieve something worth while and useful? Will the pupil's interest not be greater if everything he learns in algebra is at once used in several practical problems? Problems can and should be real and based on actual data taken by the students. Utility should not be the primary object of a student's exploration of the mathematical field. Yet it can be made a prime mover in making him want to explore it.

The ability to reason in abstract terms can best be acquired by first performing parallel reasoning processes with real tangible objects. Some mathematics laboratory work should therefore be included.

No subject is as useful a tool as mathematics or was born to meet such definite needs of the race. Is this clear to many of our students?

In organizing a new course to remedy these defects three questions must be considered:

1. How much of the present course in algebra can be made vital to the student?
2. What sacrifices must be made in subject matter and in time in order to derive the obvious benefit of selling the course to the student so that he will enjoy it?
3. How can this best be accomplished?

Briefly the plan is this. By fusing or correlating the science and mathematics recitation and laboratory material and using the science as a motivating power the entire algebra course can be justified in the eyes of the student. Yes, not only justified but also proved to be a vital necessity in the further quantitative study of the sciences. Science thus introduced would no longer be a frail skeleton of a few measured facts overlaid with the huge bulk of a loosely connected smattering of descriptive matter. A larger number of quantitative experiments performed by the instructor would give the students the true idea that science asks not only "how" and "why" but also "how

much" and "how long." Mathematics laboratory takes but a small portion of the time and consists in the measurement of real objects and situations in and outside of school. Data from both laboratories serve as problem material for the mathematics class.

The following topics are stressed. The graph, formula, functional relationships and problems in simple equations in one unknown, fractional, simultaneous, and quadratic equations, intuitive geometry, trigonometry of the right triangle, and logarithms. This is done without inserting one problem the use of which is not made apparent.

The whole course is centered around one idea, functional dependence of one variable upon another. This keeps the course from being a hash, an inhomogeneous mixture of algebra, geometry, and what not. In this respect the course is similar to a chemical compound incapable of being separated into its constituents without losing its identity and its value to the student.

The substitution of numerical trigonometry, logarithms, intuitive geometry, and other interesting work for some of the formal manipulation of algebra is no novelty. It is highly recommended by all modern educators. The inclusion of science in mathematics seems to me only a further evolution of the same idea, that these subjects are seldom encountered in practice divorced one from the other.

A supplementary reading text is used in science. The project sheets that the students daily complete in school and at home, notes taken in class and outside, frequent tests and remedial work, if the results of the tests indicate it is needed, are bound at the end of the year and are the only algebra text used.

The plan has not yet been worked out for second-year geometry. The second-year geometry was found to be aided by the preliminary work in the first year (and by the elimination of some students not fitted for the work) to such an extent that in the last half of the second year two days a week could be devoted to algebra. This gave ample time for drill work and manipulations not covered in the first year and made for more continuity throughout the high school.

This work was not designed to meet college board requirements. Strange as it may seem the revision of this work has made for success in enabling students not only to get into college but to stay there. All students who have so far passed the course have passed the college board examinations. Thus we may still maintain our cherished standards and make the work palatable as well.

Three Mathematical Songs*

Conic Sections

The first three lines should be sung at an ordinary rate, the last very fast, slowing up on the last three syllables.

(To the tune of Mistress Shady)

O Conic Sections, you've my affections,
Your rounded contours are sure to please.
Most famous loci, you have some foci,
And directrices, and asymptotes, and tangents, and subnormals,
and all kinds of eccentricities.

My predilections for Conic Sections
Were early marked by my proud papa;
Ere I could prattle or shake a rattle,
When I tumbled from my crib 'twas my habit to descend
on an arc of a parabola.

But in selections of Conic Sections
No one of course must the rest eclipse;
On this I brooded, and then concluded,
When I started every year on my journey round the sun
my path should always be an ellipse.

If these connections with Conic Sections
Some day seem boresome and dull to me,
I'll take to flying, all laws defying,
On a hyperbolic arc I will course among the stars,
trying two routes to infinity.

Sing a Song of Six Points

(The tune of "Sing a Song of Sixpence" may be found in a collection of Mother Goose Melodies)

Sing a song of six points that on a conic lie.
Join them, and a hexagon greets your ravished eye.

* These three songs were contributed by Miss Helen A. Merrill of the Department of Mathematics at Wellesley College who wrote them for the Wellesley Mathematics Club.—THE EDITOR.

If the sides are all produced, two by two they meet
On the so-called Pascal line, now isn't that quite neat?

Sing a song of Euclid, a mighty wizard he,
Once you grant his axioms you're led to Q.E.D.
Though he wrote his *Elements* in ages far and dim,
The world will always find it true, and always honor him.

Sing a song of Newton, many things he found,
Such as why an apple tumbles to the ground.
But his great discovery of the Calculus
Is what most particularly makes him dear to us.

Sing a song of Descartes with his law of signs,
And his way of drawing plus and minus lines.
He was the philosopher who was because he thought,
But 'twas to Mathematics that his greatest gifts he brought.

Sing a song of Taylor, a friend of Newton he,
Who for his work in series will long remembered be;
 $F(a + x)$ he'll teach you to express
In an infinity of terms, or, if you wish, in less.

Sing a song of all the folks, today and long ago,
Who helped discover wonders that we simply long to know.
How we'd like ourselves to find something nice and new,
To give the students after us a little more to do!

Greek and Mathematics

(To the tune of Yankee Doodle)

When I hear a crowd of folks all chattering in Greek, it
Sounds so mathematical I join right in and speak it.

Chorus

Alpha, lambda, hexagon, axiom, phi, beta,
Asymptote, pi, epsilon, theorem, rho, theta.

Of these Greek remarks I know the meaning mathematic,
Though perhaps it differs from the usage of pure Attic.

Chorus

Charles Lutwidge Dodgson (Lewis Carroll)

1832-1898

"... that peculiar love of nonsense so characteristic of the English genius. It springs only from genius as seen in the Shakespearean Fools, Lear's *Book of Nonsense*, Lewis Carroll's *Alice in Wonderland*, and the other two inspired works of that rather prim and starchy don, who vainly tried to infuse elementary mathematics into Christ Church undergraduates."—HENRY WOOD NEVINSON, *Rough Islanders*, George Routledge and Sons, Ltd., London, 1931, pp. 227.

As is well known to all who have read the classics of childhood, Lewis Carroll was the mathematician Charles Lutwidge Dodgson. As is well known to all who have read Dodgson's books on mathematics, he was the Lewis Carroll who wrote *Alice in Wonderland*, the best-known work of its kind written in the 19th century. As is well known to both readers of children's literature and of somewhat whimsical mathematical lore, the pen name which was used in *Alice in Wonderland* was formed by translating the semi-teutonic Lutwidge into Lewis, and the Charles into the semi-latin Carroll.

THE STORY goes that Queen Victoria being delighted with *Alice's Adventures in Wonderland*, sent to her bookseller for other works by the same author—but when they arrived she was astonished to find them abstruse volumes on determinants and Euclid with the name Charles Lutwidge Dodgson on the title page. It is fitting, on the hundredth anniversary of the birth of Lewis Carroll, to call to mind bits of his nonsense that show his mathematical interest and to rehearse certain events of his rather uneventful life.

Charles Lutwidge Dodgson, the eldest son of a clergyman, Charles Dodgson, was born at Daresbury, Cheshire, on January 27, 1832, and died at Guilford, Surrey, January 14, 1898. He grew up with his ten brothers and sisters in the country in the north of England. During the long holidays the rectory children apparently had few playfellows outside their own group, but they supplied that lack by games of their own contriving—a marionette show, a railroad with stations planted about the garden, and a magazine called the *Rectory Umbrella* which boasted poems, essays, and illustrations. As was customary, Lewis Carroll was sent to boarding school at the age of twelve, going first to Richmond and later to Rugby. In Janu-

ary, 1851, he began his residence at his father's college, Christ Church, Oxford. Here, on the recommendation of a friend of his father's, but also on his own merits, he was given a studentship, the conditions being that he should take orders in the church and should remain unmarried during the time he held it. His probable career was outlined by his father in a letter of August, 1855, showing how by holding his university position for ten years and living frugally in the meantime, he would be ready to accept a living—that is to take charge of a parish—with the advantage of a fair sum of ready money, a life insurance policy well started, and a library of some size. Lewis Carroll, however, failed to follow this course. He was naturally shy and retiring. He stammered and seemed to dread reading a service, and although he was made a deacon he never took priest's orders. Accordingly, instead of leaving Christ Church, he remained as a tutor and lecturer until he retired. During his early years as a tutor, he began contributing to various papers, among them *The Comic Times* and *The Train*, for which his pseudonym—Lewis Carroll—was formed, the derivation of this name being by way of Latin, Lutwidge=Ludovicus=Lewis; Charles=Carolus=Carroll.

Lewis Carroll's works show his interests ranging from amateur photography to the study of determinants. Among them are: *Notes on the First Two Books of Euclid* (1860), *The Dynamics of a Particle* (1865), *Alice's Adventures in Wonderland* (1865), *Through the Looking Glass and What Alice Found There* (1871), *Notes by an Oxford Chiel* (1874), *The Hunting of the Snark* (1876), "Doublets," *A Word Puzzle* (1879), *Euclid and His Modern Rivals* (1879), *Sylvie and Bruno* (1889), *Curiosa Mathematica*, three parts (1888-1893).

The casual references to mathematics in the two Alice's are familiar—the branches of arithmetic: "Ambition, Distraction, Uglification and Derision," the converses at the Mad Tea-Party—"I mean what I say, I say what I mean," and the others, the Red Queen's test—"Take a bone from a dog and what remains?"

Less familiar are Lewis Carroll's comments on various happenings at Oxford. *The Dynamics of a Particle* later reprinted in *Notes of an Oxford Chiel*, treated the election of the member of Parliament for Oxford from a geometric point of view. It includes definitions among which are the following:

Plain superficiality is the character of speech, in which any two points being taken, the speaker is found to lie wholly with regard to those two points.

Plain anger is the inclination of two voters to one another who meet together but whose views are not in the same direction.

When two parties coming together feel a Right Anger, each is said to be *complimentary* to the other, though, strictly speaking, this is very seldom the case.

A *surd* is a radical whose meaning cannot be exactly ascertained.

These definitions are followed by a series of propositions in which there occurs:

Proposition III. To estimate Profit and Loss. Example: Given a Derby Prophet, who has sent three different winners to three different betting men and given that none of the three horses are placed. Find the total loss incurred by the three men (a) in Money, (b) in temper. Find also the Prophet. Is this latter usually possible?

Proposition IV. The end (i.e., "the product of the extremes") justifies (i.e., "is equal to"—see Latin *aequus*) the means.

When the professor of physics submitted a statement of the needs of his department, Lewis Carroll produced similar requirements for mathematics:

A. A very large room for calculating Greatest Common Measure. To this a small one might be attached for Least Common Multiple. . . .

B. A piece of open ground for keeping Roots and practicing their extraction; it would be advisable to keep Square Roots by themselves as their corners are apt to damage others.

C. A room for reducing Fractions to their Lowest Terms. This should be provided with a cellar for keeping the Lowest Terms when found, which might be available also to the general body of Undergraduates for the purpose of "Keeping Terms."

D. A large room which might be darkened and fitted up with a magic lantern for the purpose of exhibiting circulating decimals in the act of circulation. . . .

E. A narrow strip of ground, railed off and carefully leveled, for investigating the properties of Asymptotes, and testing practically whether Parallel Lines meet or not: for this purpose, it should reach, to use the expressive language of Euclid "ever so far."

Dodgson was not a great mathematician, but his skill in logic combined with a good training in pure mathematics and an unusual ability to write in an interesting fashion, enabled him to produce a few elementary works of considerable merit.

Euclid and His Modern Rivals is a plea in dramatic form for the retention of Euclid as a text, various other treatments being examined and rejected. A typical bit is a comment on Henrici:

I haven't quite done with points yet. I find an assertion that they never jump. Do you think that arises from their having "position" which they feel might be compromised by such conduct?

This work was later reprinted in *Curiosa Mathematica*, but both forms are now quite rare.

His *Euclid and His Modern Rivals* was an excellent satire upon modern textbooks in geometry, entirely justified from the scientific standpoint but written without regard to the modern problem of teaching the general class of pupils now taking the subject. There are no usable textbooks in geometry at the present time that make any pretense to the rigor which characterizes Euclid's great classic. In fact, Book V is beyond the appreciation if not the comprehension of a rather large proportion of teachers, to say nothing of the impossibility of bringing it within the range of knowledge of more than one pupil out of a hundred.

The *Doublets* were schemes of shifting from one word to another such as were recently used in a popular magazine. For instance: Prove PRISMS to be ODIIOUS.

PRISM
PRISMATIC
DRAMATIC
MELODRAMATIC
MELODIOUS
ODIOUS

Pillow Problems which constituted part III of *Curiosa Mathematica* were drawn from many fields of mathematics. Their common characteristic was that each was supposed to be solved without recourse to pencil, paper, table of logarithms, or other aids.

These were among the works of the "rather prim and starchy don" whose favorite play fellows were children and who seemed perpetually young. Among his published letters* the most delightful are to the children. In a letter to the little girl who played "Alice" when it was first put on the stage, he begs to be excused from the "millions of hugs and kisses" that she sends, saying that millions means at least 2 and computing the time needed for 2,000,000 counting a twelve-hour day. He also refuses to be reproached for not having written "since the last letter." How could he have done so?

* See Stuart Dodgson Collingswood, *Life and Letters of Lewis Carroll*, London, 1898, and Isa Bowman, *The Story of Lewis Carroll*, New York, 1900.

With this prefatory statement, attention may be called to a very remarkable exhibition of Carrolliana to be given at Columbia University, New York, in April, 1932. This is made up of material lent from such large collections as those of Professor Zanetti of Columbia, Morris L. Parrish of Philadelphia, and Owen D. Young of New York, and from a number of other collections in different parts of the country. It includes all of the early editions of *Alice's Adventures in Wonderland*; the original manuscript of *Alice's Adventures Under Ground*, the most important manuscript of Dodgson's extant (from the library of Eldridge A. Johnson); Carroll's own uncut copy of the latter, with ten original drawings by Tenniel; eighteen original drawings by the same artist for *Alice's Adventures in Wonderland*; numerous presentation copies of Carroll's other works; letters to Alice herself (Alice Liddell, later Mrs. Hargreaves); a presentation copy to Christina Rossetti; the various editions of *Through the Looking-Glass and What Alice Found There*, one of the first editions having twenty-two original sketches by Tenniel, and another presented by the latter to a friend and containing an original sketch showing Alice passing through the mirror; a complete set of the engraver's proofs for this work, all signed by the artist; and translations of the *Wonderland* into many European and oriental languages.

Of his mathematical works the various editions are shown, including the most important of these publications such as *Euclid and His Modern Rivals*, *Curiosa Mathematica* (including his *Pillow Problems*), his *Condensation of Determinants*, and his *New Methods of Determinants*. The collection also includes all of his minor works, such as *Lawn Tennis Tournaments*, *New Methods of Scoring*, *A Tangled Tale*, *Divisibility by Seven*, *Parliamentary Representation*, and *Examples in Arithmetic*. Far more important than these minor works were his publications on logic, a subject to which he gave much attention and which led to a rather spirited contest between him and J. Cook Wilson, Professor of Logic at Oxford. Of his *Symbolic Logic* only Part I was completed; he was working on Part II at the time of his death. In the second edition of Part I he denied the story that Queen Victoria was so pleased with *Alice's Adventures in Wonderland* that she asked him to send her his other works, upon which he sent his mathematical ones. He said that nothing even resembling it had ever happened.

The exhibit contains a large amount of material relating to Carroll:

sets of games invented by him; photographs which he took; portraits of himself, including a beautiful miniature by his friend E. Gertrude Thomson; and special collections made by various admirers of his works.

The exhibit is undoubtedly the largest and most important one of Carrolliana ever assembled, and it is hardly possible that another of such magnitude will be seen elsewhere in this centenary of Carroll's birth. Its inception is due chiefly to the work of Professor Zanetti, himself a collector of such material for many years. No teacher of mathematics or lover of children's literature who lives near enough to New York, or who chances to be there during April, should miss seeing this display of exceedingly rare items relating to the life and work of a very remarkable man, one whose name is known wherever the classics of childhood are read. The catalogue itself, prepared by Professor Zanetti, is a remarkable contribution to the literature relating to Carroll, and will in time rank as one of the rare items.

It is of particular interest to all of us in America to know that the original Alice (Alice Liddell, now Mrs. Hargreaves) is to come to New York to see the exhibit. This is not only an honor to our country but a graceful tribute to the memory of the great writer whom she helped to make famous and who, in turn, made her name known throughout the world.

DAVID EUGENE SMITH
VERA SANFORD

Notice to National Council Members

Be sure to mark and return your ballot on page 51.

NEWS NOTES

Dr. George W. Myers, for nearly thirty years professor of teaching of mathematics and astronomy in the College of Education at the University of Chicago, died at his home here this morning after an illness of several months.

Dr. Myers was the author of many books on mathematics and the teaching of mathematics. He was born in Champaign County, Illinois, in 1864. He received a B.L. degree from the University of Illinois in 1888 and an M.L. in 1891. For several years Dr. Myers studied engineering at Illinois, but gave this up to study at the University of Munich, where he received a Ph.D. in 1896.

For the next three years he was director of the astronomical observatory at the University of Illinois, and in 1900 and 1901 was head of the mathematics department of the old Chicago Institute.

From 1901 to 1929 he was a member of the faculty of the University of Chicago, retiring as professor emeritus two years ago. Dr. Myers was a member of the mathematical and astronomical societies in Germany, France, Belgium, Mexico, Italy, and this country.

A widow, Mrs. Mary Sim Myers, and three children, Helen, of Des Moines, Iowa; Joseph W., of Valparaiso, Indiana; and Mrs. R. E. Lee, of Detroit, survive.

Among the mathematical textbooks prepared by Dr. Myers were *Rational Elementary and Grammar School Arithmetics*, *Myers-Brooks Elementary and Grammar School Arithmetics*,

and *Myers Arithmetics* in three volumes. He was joint author of *First Year Mathematics for Secondary Schools*, *Geometric Exercises for Algebraic Solution*, *Second Year Mathematics for Secondary Schools*, *Teachers' Manual for First Year Mathematics*, *Myers and Atwood Algebra*, and *Myers Elementary Algebraic Geometry*.

Dr. Myers was a member of Sigma Xi and Phi Beta Kappa.—*New York Times*, November 23, 1931.

THE HABIT OF THINKING

IT MAY BE CULTIVATED IN PUPILS BY
TEACHERS OF RIGHT SPIRIT

To the Editor of "*The New York Times*":

The high school teachers of New York City very laudably devoted the week beginning September 14 to consideration of the most difficult and most important of educational problems. I mean the problem of how "to develop in their pupils the power and habit of thinking."

It would be a momentous event if a formula could be found that would enable high school teachers to meet the demands of that great problem. There is, however, no reason to expect that such a formula can ever be invented. But there is in that fact no reason or justification for despair. For, although no complete solution may be reasonably expected, a little reflection suffices to show the nature of a highly important partial solution. If we cannot state fully the sufficient conditions for enabling high school teachers to develop in their pupils the power

and the habit of thinking, we may be certain that we know one, a very grave one, of the necessary conditions.

That necessary condition may be first stated in the form of a negative proposition. It is that no high school teacher can develop in his pupils the power and habit of thinking unless that power and that habit have been developed in the teacher himself. The condition stated is a necessary one, but it is obviously not sufficient. Everyone knows that a man or a woman may have the power and habit of thinking, and may have great joy in thinking, without the ability to develop such power and habit and joy in pupils. What is lacking in such exceptional cases is native aptitude for teaching or training in the art of teaching or in both. In my further discussion such aptitude and training will be assumed. That being so, we may now pass from the negative statement of the necessary condition to a positive statement of it, and say: A high school teacher in whom has been developed the power and the habit of thinking can develop that power and that habit in his pupils.

The last proposition is supported by a large variety of weighty considerations. I will briefly indicate some of them. Intellectual curiosity, or wonder, is the feeling or faculty in which, as Aristotle said long ago, all philosophical reflection begins. A teacher who has prescribed equipment of power and habit knows from personal experience not only the nature of intellectual curiosity but the kind of situations that awake and sustain it. He is thus qualified to arouse and develop that thought-engendering faculty in his pupils. Such a teacher, being one who has gained and steadily maintains the status of an independent and autono-

mous thinker, knows how to foster the growth of intellectual autonomy in his pupils; having himself acquired the fortitude to withstand the manifold influences that make against honesty and candor of thought, he will know how to engender such fortitude in his pupils, making them alert and valiant rebels against the multiform tyranny of fear; having learned the literally infinite difference between genuine conviction and mere opinion, he will lead his pupils to see that genuine conviction can come only as a result of hardy skepticism and circumspect examination of all available evidence, while mere opinions spring up like weeds and without cultivation flourish in all the fields and highways of community life.

But the greatest of all the considerations remains to be stated. It is that, no matter whether the teacher's subject be or be not one of the so-called branches of science, the teacher, if he be one who as acquired the power and the habit of thinking, will be animated and controlled by the scientific spirit. That spirit is the most precious influence in the world. For the spirit of science, the spirit of genuine thinking, is the spirit of truth-seeking. It is, therefore, both reverent and austere, demanding absolute disinterestedness, absolute fearlessness, absolute veracity, absolute freedom from prejudice, absolute freedom from taboos, absolute loyalty to the highest known standards for the ascertainment of fact. The scientific spirit, the spirit of genuine thinking, though it is thus austere, is yet a magnanimous spirit, non-provincial, non-sectarian, non-partisan, non-tribal, non-national, non-racial. It is, moreover, a friendly spirit, not selfish nor jealous nor vain nor contentious nor quarrelsome; criticism it

neither resents nor repels but invites and welcomes. It is a spirit that never resorts to war; in all times and every where throughout the world its great devotees, led by intellectual curiosity and sustained by the inner compensations of research, have been and are friendly, peaceful, co-operating rivals in the work of augmenting and spreading abroad the radiance of psychic light. The spirit of science, the spirit of genuine thinking, the spirit that seeks truth, the spirit that leads to knowledge and wisdom and understanding, is the animating principle of ideal man.

Pupils of a school where the teachers are quickened and controlled by that spirit are indeed blessed. Pupils of a school in whose teachers that spirit is lacking are indeed unfortunate, for in such pupils the power and habit of thinking will not be developed.—
CASSIUS J. KEYSER, Columbia University, September 17, 1931.

We think that the readers of THE MATHEMATICS TEACHER will be interested in the following program of the Detroit Mathematics Club for the year 1931-32:

November 19, 1931: "Standards of Scholarship in Our High Schools," J. B. Edmonson, Dean of the School of Education, University of Michigan.

February 18, 1932: "Formalism in Mathematics Teaching," E. R. Hedrick,

Professor of Mathematics, University of California at Los Angeles.

April 7, 1932: "Mathematics in the Prediction of Business Conditions," Harry C. Carver, Associate Professor of Mathematics and Insurance, University of Michigan.

May 12, 1932: "Relation of Mathematics to Business Careers," Russel V. Allman, Bond Department, Guardian Detroit Company.

"Relation of Mathematics to Vocational Careers," O. Frank Carpenter, Assistant Principal, Wilbur Wright Co-operative High School.

At these meetings, all of which are held at Northern High School, tea is served at 3:30 P.M. and the addresses are given at 4:30 P.M.

At the December meeting of Section 19 (Mathematics) of The New York Society for the Experimental Study of Education Dr. Joseph Seidlin, of Alfred University, spoke on "What the High School Mathematics Teacher Contributes to Inefficient College Teaching" and Dr. Vera Sanford, of Western Reserve University, spoke on "Arithmetic and its Perfection." On January 16, 1932, Professor Carl A. Garabedian, of St. Stephen's College, will speak on "Mathematics in Relation to Life" and on February 27, Professor Earl R. Hedrick, of the University of California, will speak on "Formalism in Mathematical Teaching."

The National Council of Teachers of Mathematics, Thirteenth Annual Meeting

Washington, D.C.

February 19, 20, 1932

The Raleigh Hotel

General Theme

MATHEMATICS IN THE ADJUSTMENT OF THE INDIVIDUAL TO CIVILIZATION

The National Council of Teachers of Mathematics is a national organization of mathematics teachers in elementary and secondary schools. Its purpose is the promotion and stimulation of better teaching of mathematics. The National Council operates chiefly through three divisions of its organization; namely, THE MATHEMATICS TEACHER, *The Year Book*, and the annual meeting of its members and board of directors.

The National Council is supported and maintained by direct membership of individuals, but desires and encourages the affiliation of other groups of teachers having kindred aims. The annual dues are \$2.00 and THE MATHEMATICS TEACHER is mailed to all members. You doubtless know someone who should join. Will you help your profession by calling attention to the Council and the value of membership?

To become a member send \$2.00 to THE MATHEMATICS TEACHER, 525 W. 120th St., New York City.

Program

The sessions of the thirteenth annual meeting of the Council, except the business session, are to be held at the Raleigh Hotel and are open to members and their friends, without any formality, but all persons are requested to register their attendance with the clerk who will be stationed near the entrance to the room where the meetings are held.

Friday Evening, February 19

8:00 P.M.

Address—Topic to be announced later.

W. C. Myers, Asst. Principal McKinley Manual Training School,
Washington D.C.

Address—"A Comparative Study of the Teaching of Mathematics
in the United States and Germany."

W. D. Reeve, Professor of Mathematics, Teachers College, Co-
lumbia University, and Editor of *THE MATHEMATICS TEACHER*.

Saturday, February 20

7:30 A.M.

Breakfast and Advisory Committee Meeting of Delegates from
Affiliated Organizations and Officers of the National Council of Teach-
ers of Mathematics.

9:00 A.M.

Bureau of Standards Lecture Room, Fourth Floor

Annual Business Meeting, including election of officers and the re-
port of the Secretary-Treasurer.

10:15 A.M.

Visit to the Bureau of Standards, Connecticut Avenue and Upton
Street.

This privilege is extended to members of the Council through the
courtesy of Mr. George K. Burgess, Director of the Bureau of Stand-
ards.

Saturday, February 20

2:00 P.M.

"Improving America's Mathematics,"

Harry C. Barber, Department of Mathematics, Phillips Exeter
Academy, Exeter, New Hampshire.

"What Do We Owe to the Brighter Pupil?"

Miss Beulah I. Shoesmith, head of the Department of Mathe-
matics, Hyde Park High School, Chicago, Illinois.

Report of a Study of Extra-mural Examinations,

W. S. Schlauch, Chairman of the Standing Committee of the

Council on Cooperation with Official Examining Boards, and
Professor of Education, New York University, New York City.

Saturday, February 20

6:30 P.M.

BANQUET

Address—"What Mathematics Means to the World," E. R. Hedrick,
Chairman of the American Committee of the International Commission
on the Teaching of Mathematics, and Professor of Mathematics, Uni-
versity of California at Los Angeles.

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The local committee for Washington are:
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George A. Ross, Central High School.

William C. Myers, McKinley Manual Training School.

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SPECIAL NOTICE: The annual meeting of the Board of Directors of the Council will be held at the Raleigh, beginning Friday, February 19, at 10:00 A.M. and continuing during the afternoon.

TEACHING HELPS in Geometry

WHY are there so many teaching helps in the Strader and Rhoads GEOMETRIES? Because the authors themselves are classroom teachers and they know what teachers want. . . . Exercises are graded and classified for oral, diagnostic, remedial, sight and supplementary work, special drill and review. Minimum courses are definitely outlined. Explanations permit and encourage outside preparation. Sequence is flexible. Geometric figures are plentiful and clearly drawn. Striking illustrations aid motivation. Teachers' Manuals* are complete and helpful. Examination copies sent on request.

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of The National Council of Teachers of Mathematics.

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